647510 **MSC4MatC404x** **Seat No :_____**

M.Sc.(Maths) Semester - 4 (*CBCS*) Examination
May/June-2021 (NEW COURSE)
Graph Theory (CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1

A Verify by example that Adjacency matrices of isomorphic graphs are similar. **4**

B **Attempt Any two** **10**

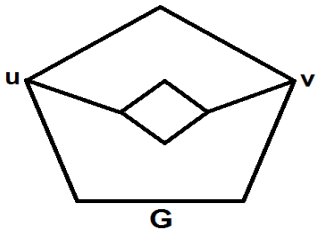
(1) Define Subgraph. Explain Various types of Sub graph of a Graph.

(2) State and prove Fundamental theorem of Graph Theory.

(3) Define Degree Sequence of Graph. State and prove necessary and sufficient condition of a non-negative sequence to be degree sequence.

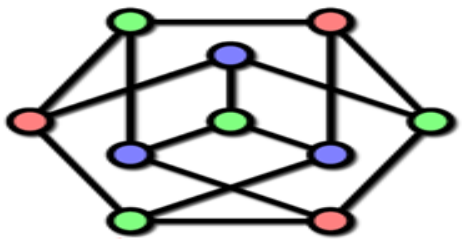
Q.2

A Define Adjacency matrix A (G) and Path matrix P(*u*, *v*) (between vertices *u* & *v*)of graph G. **4**
Find A (G) & P(*u*, *v*) for the following graph.



B **Attempt Any two** **10**

(1) Find Distance, Diameter, radius & eccentricities of each vertex of the following graph.



(2) State and Prove Euler Theorem

(3) Construct the following digraph & Find the following.
$D = (V, E); V = \{a, b, \dots, i\}$&
$E = \{ ba, cb, dc, dd, ed, ce, ec, fe, hf, ch, ic, hi, bi, ia, ig, ag, gf, ag, gh \}$
Find: (1) Self loop and Multi arcs (2) In degree and Out Degree & Verify First Theorem of Digraph (3) Open walk & Closed Trail(4) Open walk but not open trail.

Q.3

A If G is a simple graph with n vertices and k components, prove that G has at least n – k number of edges. **4**

B **Attempt Any two** **10**

(1) Show that K_5 & $K_{3,3}$ are non- planner graphs.

(2) Show that the Petersen graph is non planar.

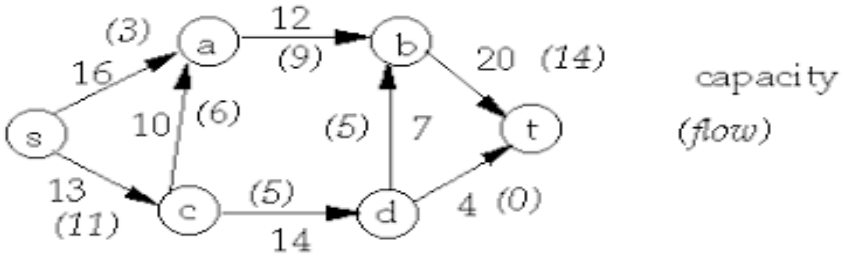
(3) Show that if a planar graph G of order n and size m has r regions and K components, then n – m + r = k + 1.

Q.4

A A connected graph has nine vertices having degrees 2, 2, 2, 3, 3, 3, 4, 4 and 5. How many edges are there? How many faces are there? **4**

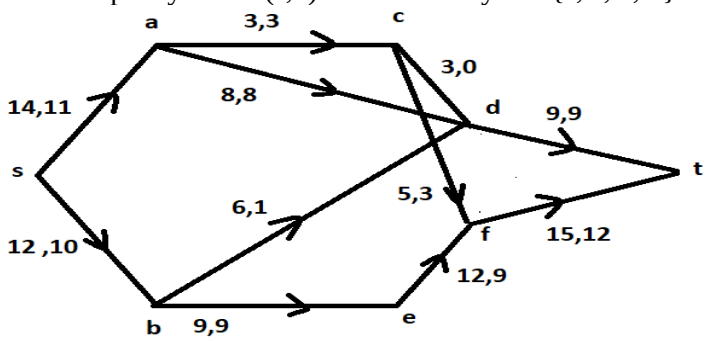
B **Attempt Any two** **10**

(1) Define Flow of a Network. Verify the laws of Flow in the given transport network with flow and find the value of the flow.



(2) Show that In any network, the value of any maximum flow is equal to the capacity of any minimum cut.

(3) Find the capacity of the (s, t)-cut defined by S = {s, a, b, d} and T = {c, e, f, t}.



Q.5

A Define Vertex Connectivity and Edge Connectivity of a graph. Find these parameters for the graph K_n & $K_{m,n}$ **4**

B **Attempt Any two** **10**

(1) Show that every connected graph has a spanning tree.

(2) A graph of n vertices is a complete graph if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \ldots\ldots (\lambda - n + 1)$

(3) A graph G is 2-chromatic if and only if it is a non-null bipartite graph.

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